

Students explore inverses, that is, equations that “undo” the actions of functions. Although they may not be aware of it, students have already seen inverses without calling them that. When solving equations, students reverse operations to undo the equation leaving just  $x$ , and that is what inverses do. Students further explore composition of functions by considering what happens when inverses are combined. For further information see the Math Notes boxes following problems 6-25 on page 269, 6-43 on page 274, and 6-50 on page 276.

### Example 1

Find the inverse (undo) rule for the functions below. Use function notation and give the inverse rule a name different from the original function.

a.  $f(x) = \frac{x-6}{3}$

b.  $g(x) = (x + 4)^2 + 1$

The function in part (a) subtracts six from the input then divides by three. The undo rule, or inverse, reverses this process. Therefore, the inverse first multiplies by three then adds six. If we call this inverse  $h(x)$ , we can write  $h(x) = 3x + 6$ .

The function  $g(x)$  adds four to the input, squares that value, then adds one. The inverse will first subtract one, take the square root then subtract four. Calling this rule  $j(x)$  we can write  $j(x) = \pm\sqrt{x-1} - 4$ .

Rather than give the inverse a new name, we can use the notation for inverses. The inverse of  $f(x)$  is written as  $f^{-1}(x)$ .

Note: The inverses  $h(x)$  and  $j(x)$ , are fundamentally different.  $h(x) = 3x + 6$  is the equation of a non-vertical line, therefore  $h(x)$  is a function.  $j(x) = \pm\sqrt{x-1} - 4$  however, is not a function. By taking the square root, we created a positive value and a negative value. This gives two outputs for each input, and so by definition it is not a function.

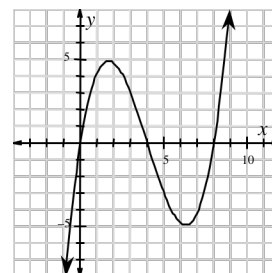
Although we computed the inverses through a verbal description of what each function does, the students learn an algorithm for finding an inverse. They switch the  $x$  and  $y$ , and then solve for  $y$ . Using this algorithm on the equations above:

$$\begin{aligned} f(x) &= \frac{x-6}{3} \\ y &= \frac{x-6}{3} \\ x &= \frac{y-6}{3} \\ 3x &= y-6 \\ 3x+6 &= y \end{aligned}$$

$$\begin{aligned} g(x) &= (x+4)^2 + 1 \\ y &= (x+4)^2 + 1 \\ x &= (y+4)^2 + 1 \\ x-1 &= (y+4)^2 \\ \pm\sqrt{x-1} &= y+4 \\ -4 \pm \sqrt{x-1} &= y \end{aligned}$$

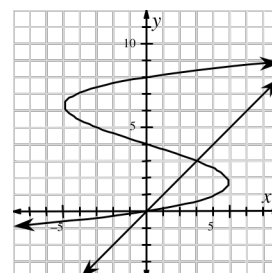
## Example 2

The graph of  $f(x) = 0.2x^3 - 2.4x^2 + 6.4x$  is shown at right. Graph the inverse of this function.



Following the algorithm for determining the equation for an inverse, as we did above, would be difficult here. The students do not have a method for solving cubic equations. Nevertheless, students can graph the inverse because they know a special property about the graphs of functions and their inverses: they are symmetrical about the line  $y = x$ .

If we add the line  $y = x$  to the graph, the inverse is the reflection across this line. Here we can fold the paper along the line  $y = x$ , and trace the result to create the reflection.



## Example 3

Consider the function  $f(x) = \frac{2x-1}{7}$ . Determine the inverse of  $f(x)$  and label it  $g(x)$ . Verify that these two functions are inverses by calculating  $f(g(x))$  and  $g(f(x))$ .

Using the algorithm, we can determine the inverse.

$y = \frac{2x-1}{7}$	Composing the two functions $f(x)$ and $g(x)$ gives a method for checking whether or not functions are inverses of each other. Since one function “undoes” the other, when the functions are composed, the output should be $x$ .	
$x = \frac{2y-1}{7}$		
$7x = 2y - 1$		
$7x + 1 = 2y$		
$y = \frac{7x+1}{2}$	$f(x) = \frac{2x-1}{7}$	$g(x) = \frac{7x+1}{2}$
$g(x) = \frac{7x+1}{2}$	$f(g(x)) = f\left(\frac{7x+1}{2}\right) = \frac{2\left(\frac{7x+1}{2}\right)-1}{7} = \frac{7x+1-1}{7} = \frac{7x}{7} = x$	
	$g(f(x)) = g\left(\frac{2x-1}{7}\right) = \frac{7\left(\frac{2x-1}{7}\right)+1}{2} = \frac{2x-1+1}{2} = \frac{2x}{2} = x$	

Since  $f(g(x)) = g(f(x)) = x$ , the functions are inverses.

## Problems

Find the inverse of each of the following functions.

1.  $f(x) = 8(x - 13)$

2.  $y = -\frac{3}{4}x + 6$

3.  $y = \frac{5(x+2)}{3}$

4.  $f(x) = x^2 + 6$

5.  $f(x) = \frac{3}{x} + 6$

6.  $g(x) = \frac{5}{x}$

7.  $g(x) = (x + 1)^2 - 3$

8.  $y = (x + 2)^3$

9.  $y = 3 + \sqrt{x - 4}$

10.  $g(x) = 6x + 2$

Sketch the graph of the inverse of each of the following functions.

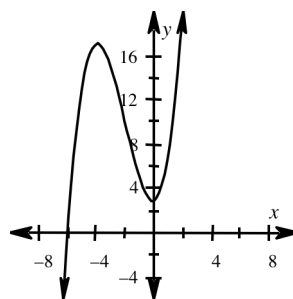
11.  $y = \frac{x}{6} + 2$

15.

12.  $f(x) = 2x^2 - 1$

13.  $g(x) = x$

14.  $y = \frac{1}{5x}$



For each of the following pairs of functions, determine  $f(g(x))$  and  $g(f(x))$ , then use the result to decide whether or not  $f(x)$  and  $g(x)$  are inverses of each other.

16.  $f(x) = 5x + 7$   
 $g(x) = \frac{x-7}{5}$

17.  $f(x) = 8x$   
 $g(x) = \frac{1}{8}x$

18.  $f(x) = x + 5$   
 $g(x) = \frac{1}{x+5}$

19.  $f(x) = \frac{2}{3x}$   
 $g(x) = \frac{3x}{2}$

20.  $f(x) = \frac{2}{3}x + 6$   
 $g(x) = \frac{3(x-6)}{2}$

21.  $f(x) = x\sqrt{3} + 9$   
 $g(x) = \left(\frac{x-9}{\sqrt{3}}\right)^2$

## Answers

1.  $y = \frac{x}{8} + 13$

6.  $y = \frac{5}{x}$

2.  $y = -\frac{4}{3}x + 8$

7.  $y = -1 \pm \sqrt{x+3}$

3.  $y = \frac{3}{5}x - 2$

8.  $y = -2 + \sqrt[3]{x}$

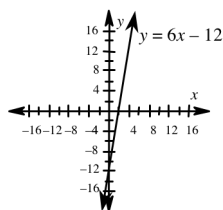
4.  $y = \pm\sqrt{x-6}$

9.  $y = (x-3)^2 + 4$ , for  $x \geq 3$

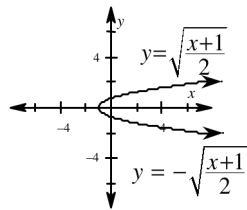
5.  $y = \frac{3}{x-6}$

10.  $y = \frac{x-2}{6}$

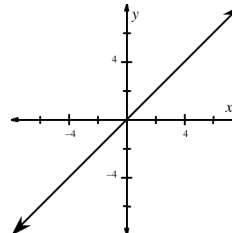
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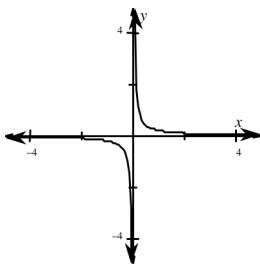
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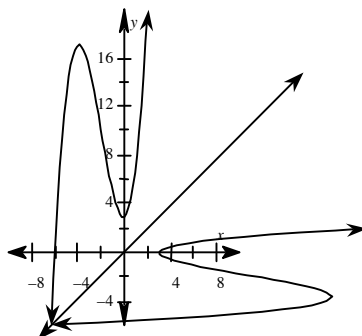
13.



14.



15.



16.  $f(g(x)) = g(f(x)) = x$ . They are inverses.

17.  $f(g(x)) = g(f(x)) = x$ . They are inverses.

18.  $f(g(x)) = \frac{1}{x+5} + 5$ ,  $g(f(x)) = \frac{1}{x+10}$ . No they are not inverses.

19.  $f(g(x)) = \frac{4}{9x}$ ,  $g(f(x)) = \frac{1}{x}$ . No they are not inverses.

20.  $f(g(x)) = g(f(x)) = x$ . They are inverses.

21.  $f(g(x)) = \frac{\sqrt{3}(x-9)^2}{3} + 9$ ,  $g(f(x)) = x^2$ . No they are not inverses.

The earlier sections of this chapter gave students many opportunities to find the inverses of various functions. Here, students explore the inverse of an exponential function. Although they can graph the inverse by reflecting the graph of an exponential function across the line  $y = x$ , they cannot write the equation of this new function. Writing the equation requires the introduction of a new function, the logarithm. Students explore the properties and graphs of logarithms, and in a later chapter use them to solve equations of this type. For further information see the Math Notes box following problem 6-71 on page 283.

### Example 1

Find each of the values below and then justify your answer by writing the equivalent exponential form.

a.  $\log_5 25 = ?$

b.  $\log_7 ? = 3$

c.  $\log_2 \left( \frac{1}{8} \right) = ?$

A logarithm is really just an exponent, so an expression like the one in part (a),  $\log_5 25$ , is asking “What exponent can I raise the base 5 to, to get 25?” We can translate this question into an equation:  $5^? = 25$ . By phrasing it this way, the answer is more apparent: 2. This is true because  $5^2 = 25$ .

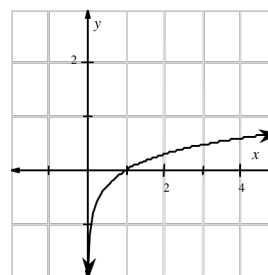
Part (b) can be rephrased as  $7^3 = ?$ . The answer is 343.

Part (c) asks “2 to what exponent gives  $\frac{1}{8}$ ?” or  $2^? = \frac{1}{8}$ . The answer is  $-3$  because  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ .

### Example 2

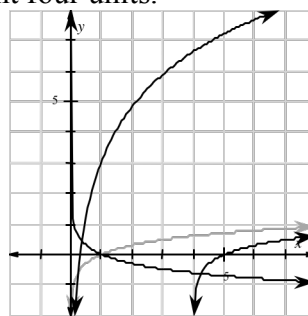
The graph of  $y = \log x$  is shown at right. Use this “parent graph” to graph each of the following equations. Explain how you get your new graphs.

$$y = \log(x - 4) \quad y = 6\log(x) + 3 \quad y = -\log x$$



The logarithm function follows the same rules for transforming its graphs as other functions we have used. The first equation shifts the original graph to the right four units.

The graph of the second equation is shifted up three units (because of the “+ 3”) but is also stretched because it is multiplied by six. The third function is flipped across the  $x$ -axis. All three of these graphs are shown at right. The original function  $y = \log x$  is also there, in light gray. Note: When a logarithm is written without a base, as in  $y = \log x$  and the log key used on a calculator, the base is 10.



## Problems

Rewrite each logarithmic equation as an exponential equation and vice versa.

1.  $y = \log_4 x$

2.  $3 = \log_2 x$

3.  $x = \log_5 30$

4.  $4^x = 80$

5.  $\left(\frac{1}{2}\right)^x = 64$

6.  $x^3 = 343$

7.  $5^x = \frac{1}{125}$

8.  $\log_x 32 = y$

9.  $11^3 = x$

10.  $-4 = \log_x \left(\frac{1}{16}\right)$

What is the value of  $x$  in each equation below? If necessary, rewrite the expression in the equivalent exponential equation to verify your answer.

11.  $4 = \log_5 x$

12.  $2 = \log_9 x$

13.  $9 = \log x$

14.  $81 = 9^x$

15.  $\left(\frac{1}{3}\right)^x = 243$

16.  $6^x = 7776$

17.  $7^x = \frac{1}{49}$

18.  $\log_2 32 = x$

19.  $\log_{11} x = 3$

20.  $\log_5 \left(\frac{1}{125}\right) = x$

Graph each of the following equations.

21.  $y = \log(x + 2)$

22.  $y = -5 + \log x$

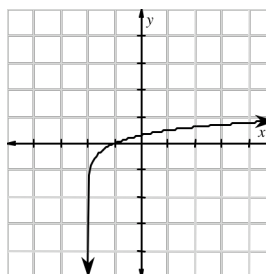
23.  $y = -\log(x - 4)$

24.  $y = 5 + 3\log(x - 7)$

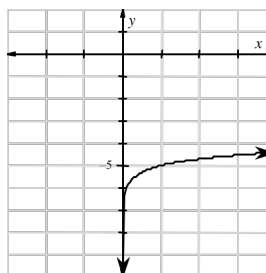
## Answers

1.  $4^y = x$
2.  $2^3 = x$
3.  $5^x = 30$
4.  $\log_4 80 = x$
5.  $\log_{1/2} 64 = x$
6.  $\log_x 343 = 3$
7.  $\log_5 \left( \frac{1}{125} \right) = x$
8.  $x^y = 32$
9.  $\log_{11} x = 3$
10.  $x^{-4} = \frac{1}{16}$
11.  $x = 625$
12.  $x = 81$
13.  $x = 1,000,000,000$
14.  $x = 2$
15.  $x = -5$
16.  $x = 5$
17.  $x = -2$
18.  $x = 5$
19.  $x = 1,331$
20.  $x = -3$

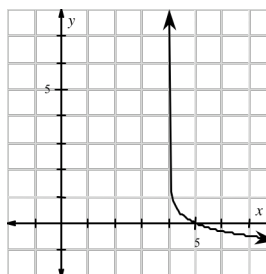
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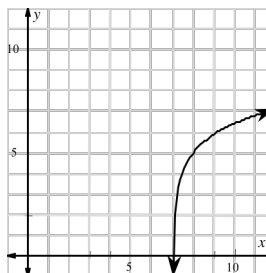
22.



23.



24.



## CHECKPOINT PRACTICE PROBLEMS

Starting in Chapter 2, several problems are marked with the icon shown at right. This icon indicates a “Checkpoint” problem for an Algebra 1 topics that students should be able to solve correctly at this point in the course. If the student needs help to solve this kind of problem, or cannot consistently solve them correctly, then the student needs additional practice with this type of problem. After each Checkpoint, the student will be expected to solve that type of problem easily and accurately.



The Checkpoint problem for Chapter 6 is problem 6-48 (integral and rational exponents). The practice problems below cover this one topic.

Use integer or rational exponents to write each of the following expression as an exponential expression with a base of  $x$ . (problem 6-48)

- |                  |                           |                    |                                       |
|------------------|---------------------------|--------------------|---------------------------------------|
| 1. $\sqrt[3]{x}$ | 2. $\frac{1}{\sqrt{x}}$   | 3. $\sqrt[3]{x^7}$ | 4. $\frac{1}{\sqrt[4]{x}}$            |
| 5. $\sqrt{x^5}$  | 6. $\frac{x^3}{\sqrt{x}}$ | 7. $\sqrt[3]{x^4}$ | 8. $\frac{\sqrt[3]{x^5}}{\sqrt{x^3}}$ |

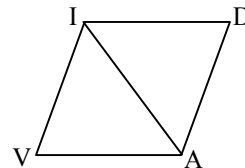
### Answers

- |               |                                |
|---------------|--------------------------------|
| 1. $x^{1/3}$  | 5. $x^{1/5}$                   |
| 2. $x^{-1/2}$ | 6. $x^3x^{-1/2} = x^{5/2}$     |
| 3. $x^{7/3}$  | 7. $x^{4/3}$                   |
| 4. $x^{-1/4}$ | 8. $x^{5/3}x^{-3/2} = x^{1/6}$ |



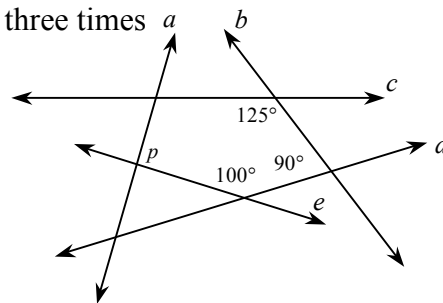
# SAT PREP

1. In the figure at right,  $\triangle DAI$  is isosceles with  $DI = 13$  and base 24. If  $\triangle VAI \cong \triangle DAI$  what is the area of quadrilateral  $DAVI$ ?



- a. 60      b. 75      c. 120      d. 156      e. 240
2. An experimental jet flies at a speed of 5280 miles per hour. How many miles can this jet cover in 10 seconds?
- a. 1.467      b. 8.802      c. 11.237      d. 14.667      e. 88.022

3. If the angle (not shown) where  $a$  and  $b$  intersect is three times as large as the angle (not shown) where  $e$  and  $b$  intersect, what is the value of  $p$ ?



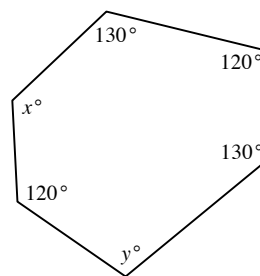
- a.  $70^\circ$       b.  $85^\circ$       c.  $140^\circ$
- d.  $160^\circ$       e. Cannot determine
4. Let  $\zeta x \zeta$  be defined for all positive integer values of  $x$  as the product of all even factors of  $4x$ . For example,  $\zeta 3 \zeta = 12 \times 6 \times 4 \times 2 = 576$ . What is the value of  $\zeta 5 \zeta$ ?
- a. 1600      b. 6400      c. 7200      d. 8000      e. 9600

5. The chart at right shows the distribution of topics covered in a particular business text, in chapters and pages per chapter. According to the chart, how many total pages are in this text?

Topic	No. of Chapters	No. of pages
Development	3	12
Marketing	4	8
Public Relations	1	11

- a. 31      b. 39      c. 48      d. 65      e. 79

6. In the figure at right, what is the sum of  $x$  and  $y$ ?  
Note: The figure is not drawn to scale.



7. If  $2^q = 8^{q-1}$ , then  $q = ?$
8. If  $a$  is 40 percent of 300,  $b$  is 40 percent of  $a$ , and  $c$  is 25 percent of  $b$ , what is  $a + b + c$ ?
9. If  $\frac{x}{4} = \frac{11}{20}$ , what is the value of  $x$ ?
10. If  $\frac{3}{5}$  of  $\frac{1}{3}$  is added to 5, what is the answer?

## Answers

1. C
2. D
3. C
4. A
5. E
6.  $220^\circ$
7.  $q = \frac{3}{2}$
8. 180
9.  $x = 2.2$
10.  $5\frac{1}{5}$